

TMA4170 Fourier Analysis

Convolutions

$$S_N(f) = (f * D_N)$$

- f, g L -periodic : $(f * g)(x) := \frac{1}{L} \int_0^L f(y) g(x-y) dy$
- Well-defined if f, g Riemann integrable (or in $L^1(\mathbb{R})$)

$$\|f * g\|_1 \leq \frac{1}{2\pi} \|f\|_1 \cdot \|g\|_1 \quad [\text{Young: } \|f * g\|_p \leq c \|f\|_1 \cdot \|g\|_p, p \in [1, \infty]]$$

- Properties :

$$(a) \quad f * (A g + B h) = A (f * g) + B (f * h) ; \quad A, B \in \mathbb{C}$$

$$(b) \quad f * g = g * f$$

$$(c) \quad (f * g) * h = f * (g * h)$$

$$(d) \quad \widehat{f * g}(n) = \hat{f}(n) \cdot \hat{g}(n)$$

$$(e) \quad f * g \text{ is uniformly continuous if } g \text{ bounded}$$